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Stable Oscillations in a Bistable, Bidirectional CO_2 Ring Laser

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Stable oscillations in a bistable, bidirectional CO_2 ring laser

By J. R. Tredicce, G. L. Lippi, F. T. Arecchi and N. B. Abraham†
Istituto Nazionale di Ottica, Largo E. Fermi 6, 50125 *Firenze, Italy*

We show experimentally and theoretically the existence of spontaneous self-modulation and mode-switching in a bidirectional ring laser. The process involves at least two different characteristic frequencies. Experimental records of the temporal behaviour and associated power spectra for various operating conditions are shown.

Introduction

The dynamical behaviour of bidirectional ring dye lasers has recently been studied (Roy & Mandel 1980*a*, *b*; Mandel *et al.* 1981). In these cases the intensity output was stable (as one mode quenched the other). Stochastic switching between the dominance of the two counter-propagating modes was observed and explained by additive and multiplicative noise (Lett *et al.* 1981).

In the presence of an external back-reflecting mirror, regular pulsations have also been observed in such a laser (Kühlke 1982). However, in a dye laser only the field variables play a relevant role in the dynamics.

In contrast, we have shown in previous work that the population inversion must be taken into account to describe the dynamical behaviour of a CO_2 laser (Arecchi *et al.* 1982, 1984). The interaction between the two fields and the population inversion leads to spontaneous pulsations in a bidirectional CO_2 ring laser.

Experimental results

The experimental arrangement is shown in figure 1. The parameters to be changed are pressure, current, cavity tuning, and tuning of the external mirror. The results reported here are for the arrangement without the external mirror. For a wide range of pressure and current, we have the previously observed bistable behaviour between two stable states, in which one mode has a constant, non-zero intensity while the other is completely extinguished. For lower pumping rates (by changing current or pressure) an unstable state appears in which a regular spiking in both modes simultaneously is possible (figure 2*a*).

For lower pressures the giant spikes are followed by damped relaxation oscillations, as shown in figure 2*b*. The spikes occur at regular intervals with a repetition rate of the order of the slow population relaxation rate. The damped oscillations can also be observed during the transients in single-mode lasers and their frequency is of the order of the geometric mean of the population and field relaxation rates. The repetition rate of the giant pulses seems to be related to the population relaxation rate. The power spectra in these two cases show a sharp peak at a low

† Present address: Department of Physics, Bryn Mawr College, Bryn Mawr, Pennsylvania 19010, U.S.A.

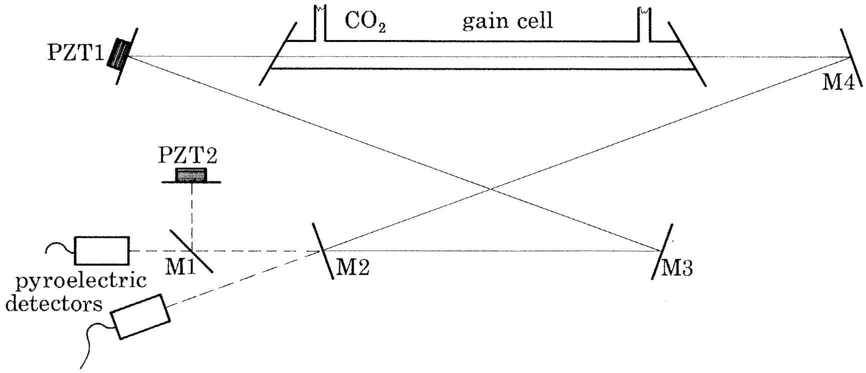

FIGURE 1. Experimental arrangement showing bidirectional CO_2 ring laser (10.6 μm wavelength) with optional external back-reflecting mirror. The total 'round-trip' cavity length is 4.2 m. The reflectivity and R for each is: M1, 80 %, ∞; M2, 80 %, 5 m; M3, 100 %, 5 m; M4, 100 %, 2 m; PZT1, 100 %, 2 m; PZT2, 100 %, 5 m.

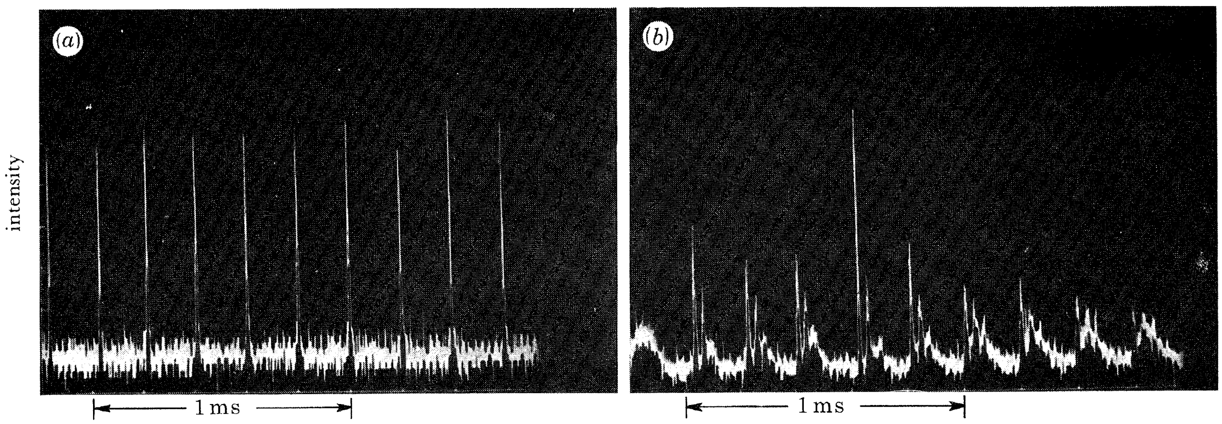

FIGURE 2. Intensity against time for one of the two modes: (a) current $I = 4$ mA per division, pressure $P = 1330$ Pa; (b) current $I = 4$ mA per division, pressure $P = 933$ Pa.

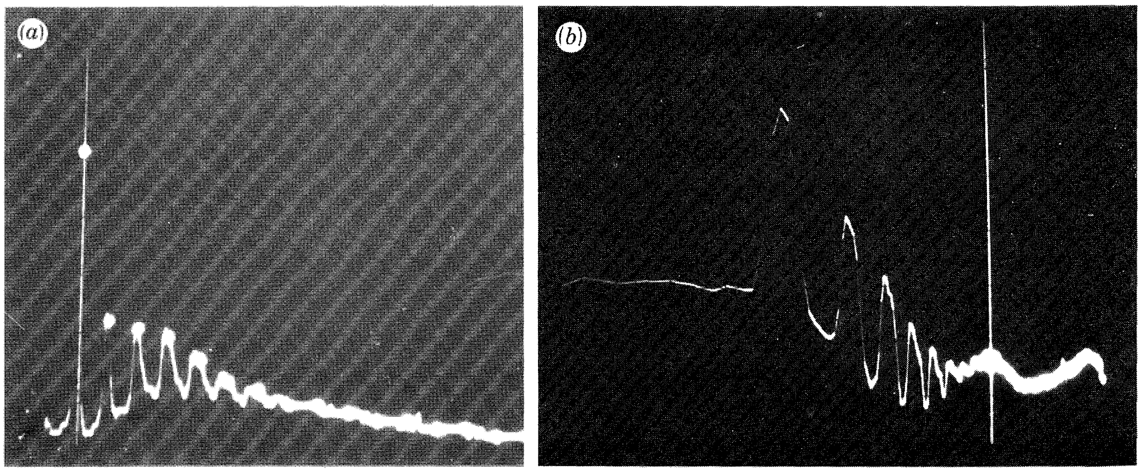

FIGURE 3. Intensity–power spectra for the current and pressure conditions of figure 2, where a and b correspond in the two cases. The frequency scale in a is plotted linearly, while in b it is logarithmic, as can be seen from the spacing of successive harmonics.

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frequency (of order 2 to 7 kHz) and peaks at its harmonics, and a second broad peak at the higher frequencies corresponding to the relaxation oscillations (figure 3*a*, *b*).

In figure 4 we are able to compare the behaviour of the two modes. In figure 4*a* the modes pulse synchronously, but their relaxation oscillations are 180° out of phase, which indicates a common origin of the principal pulsation and some type of competition in the damped ringing. In figure 4*b* we observe the switch-on of one mode (upper trace) and the switch-off the second mode (lower trace) again with out-of-phase higher frequency oscillation in the modes.

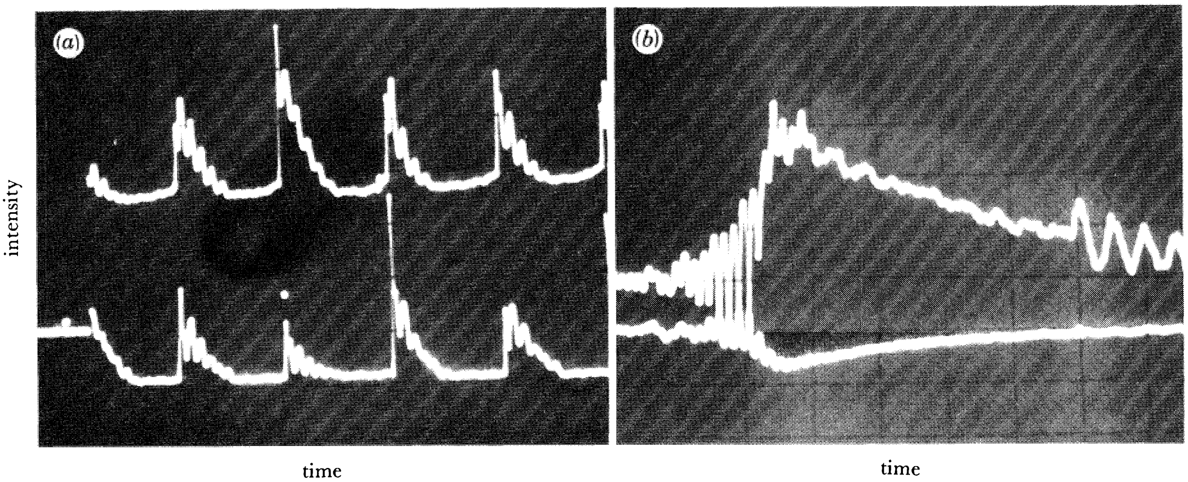

FIGURE 4. Intensity against time to show the two-mode behaviour for two different values of the cavity tuning: (*a*) synchronous pulsing with out-of-phase relaxation oscillations; (*b*) the mode switching with one on and one off.

THEORETICAL RESULTS

Theoretically, the system can be described by the Maxwell–Bloch type of equations for a two-level system interacting with two counter-propagating waves having the same spatial mode structure. By assuming that the spatial harmonic patterns formed by the two interfering modes are relatively weak, we can truncate the equations to the normal single-mode set with only lowest order corrections. Adiabatically eliminating the polarization, we arrive at a set of seven equations similar to those derived elsewhere when back-scattering was included (Perevedentseva *et al.* 1980). Here, in contrast, we retain the effects of detuning. (Previous analyses by Zhelnov *et al.* (1970) and by Mandel & Agrawal (1982) have shown that without detuning or other coupling mechanisms the asymmetric stationary states are stable.) Taking into account the complex electric fields of the two modes, the population inversion, and the complex amplitude of the spatial population grating formed in the medium by the interference of the counter-propagating fields, the equation set may be written as

$$\dot{x} = (1+\mathrm{i}\delta)^{-1}(zx+w^*y)-x,$$

$$\dot{y} = (1+\mathrm{i}\delta)^{-1}(zy+wx)\,(k_2/k_1)\,y+xR\exp(\mathrm{i}\phi),$$

$$(k_1/\gamma_\parallel)\dot{z} = -(z-z_0)-(1+\delta^2)^{-1}\{z(|x|^2+|y|^2)+w^*x^*y+wxy^*\},$$

$$(k_1/\gamma_\parallel)\dot{w} = -w-(1+\delta^2)^{-1}[zx^*y+\tfrac{1}{2}w\{|x|^2+|y|^2+\mathrm{i}\delta(|y|^2-|x|^2)\}],$$

where x and y are the complex amplitudes of the two modes, respectively, z is the population inversion and w is the complex amplitude of the population grating; δ is the detuning between

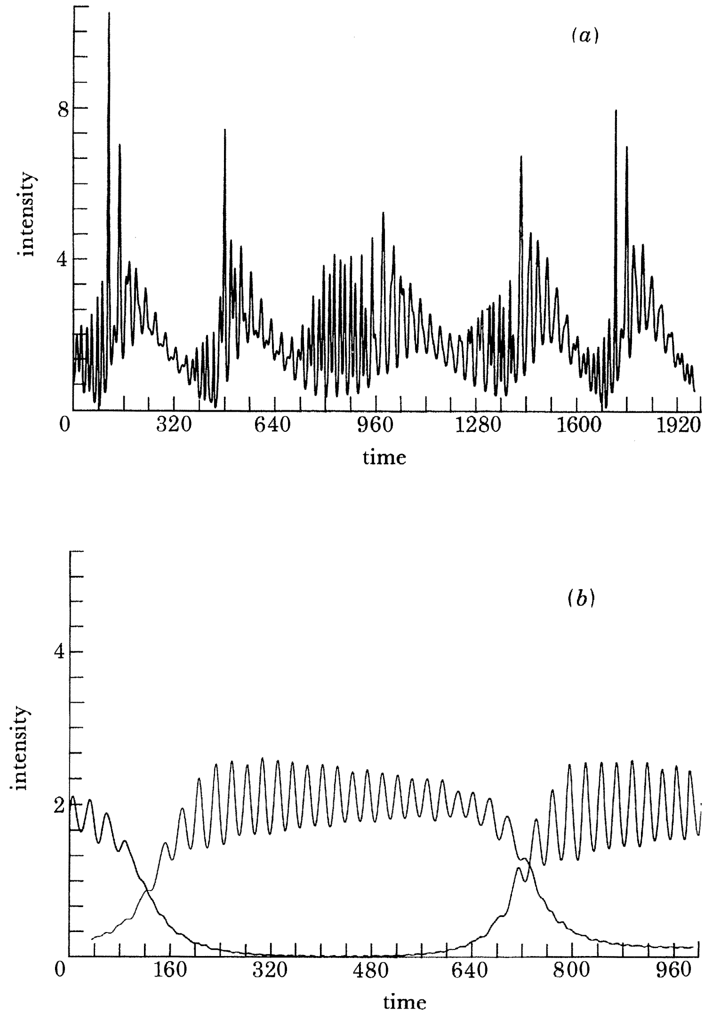

FIGURE 5. Results from numerical integration of the seven-equation model. Normalized intensity against dimensionless time (units of k_1^{-1}) for typical CO_2 laser parameters. Characteristic features of the experimental results shown in figure 4a and 4b are reproduced in (a) and (b), respectively.

the cavity and atomic frequencies, normalized to the polarization relaxation rate. R and ϕ are the amplitude and phase of the back reflection; k_1 and k_2 are the loss rates for the two modes and $\gamma_\parallel$ is the loss rate for the population inversion. Time has been renormalized to $\tau = k_1 t$ and the dot signifies differentiation with respect to τ. The parameters x, y, z and w are the slowly varying amplitudes of the rapidly varying carrier wave at a frequency equal to the atomic resonance frequency. Finally, z_0 is the equilibrium population inversion in the absence of the two fields.

Principal theoretical results include the coexistence of the two stable stationary states for zero detuning, which are destabilized with detuning in favour of spontaneous switching between them. The dynamical behaviour, when the system is unstable, is in good agreement with the experimental results, showing both simultaneous pulsing and alternate switching of the two modes (figure 5) for no back-reflection ($R = 0$), and suitable detuning.

Since completing this work we have observed similar solutions in a recent paper by Polushkin

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et al. (1983), which uses rather more complicated models for the material variables and the field coupling.

Quasiperiodic and chaotic behaviour, observed in our experimental and theoretical results, will be described elsewhere.

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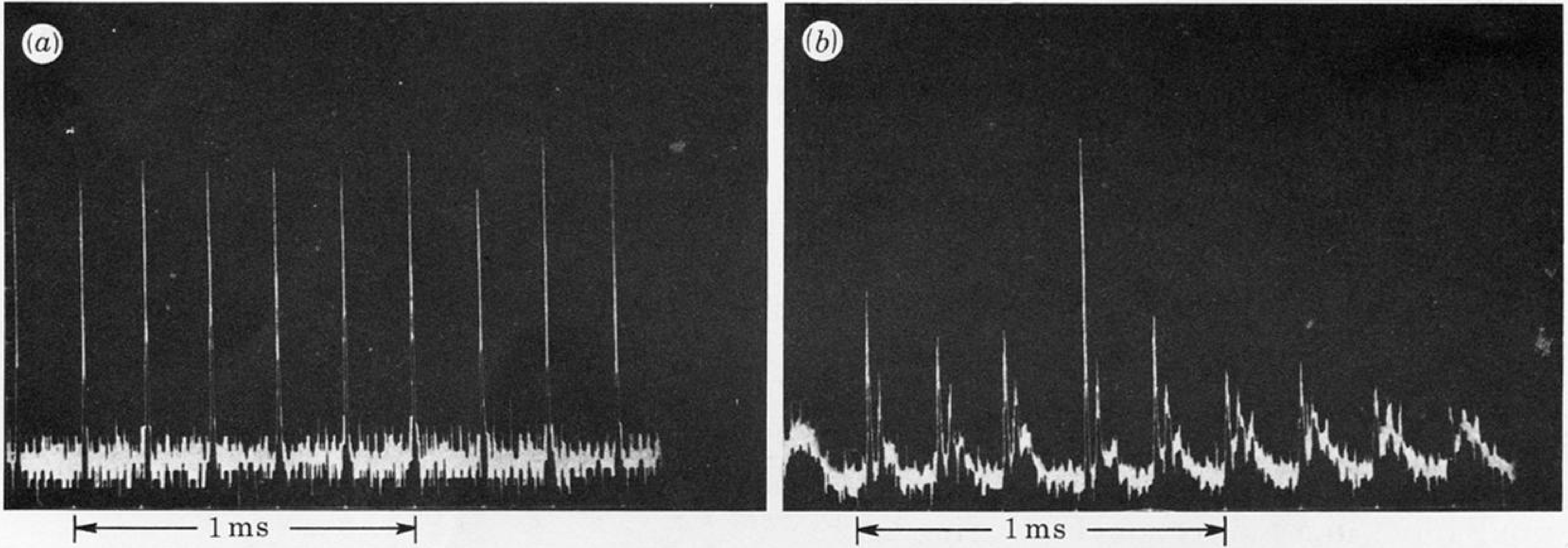

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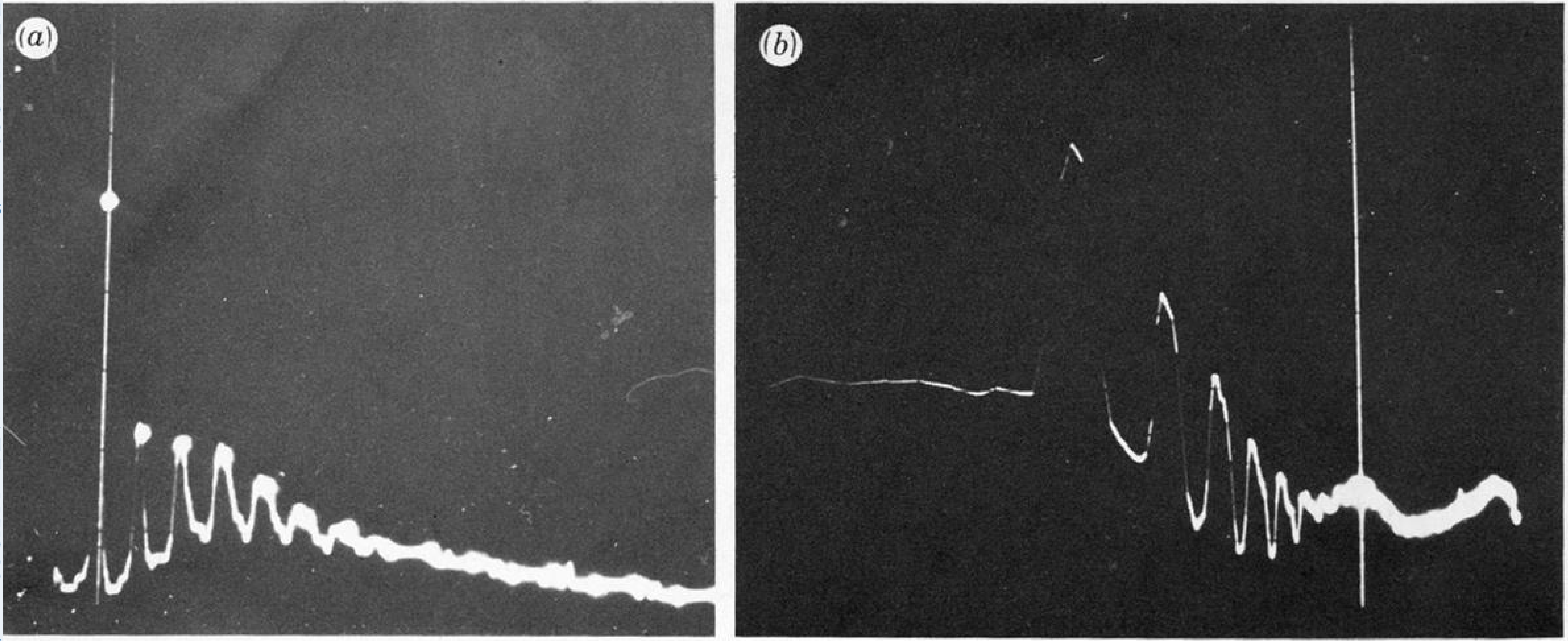

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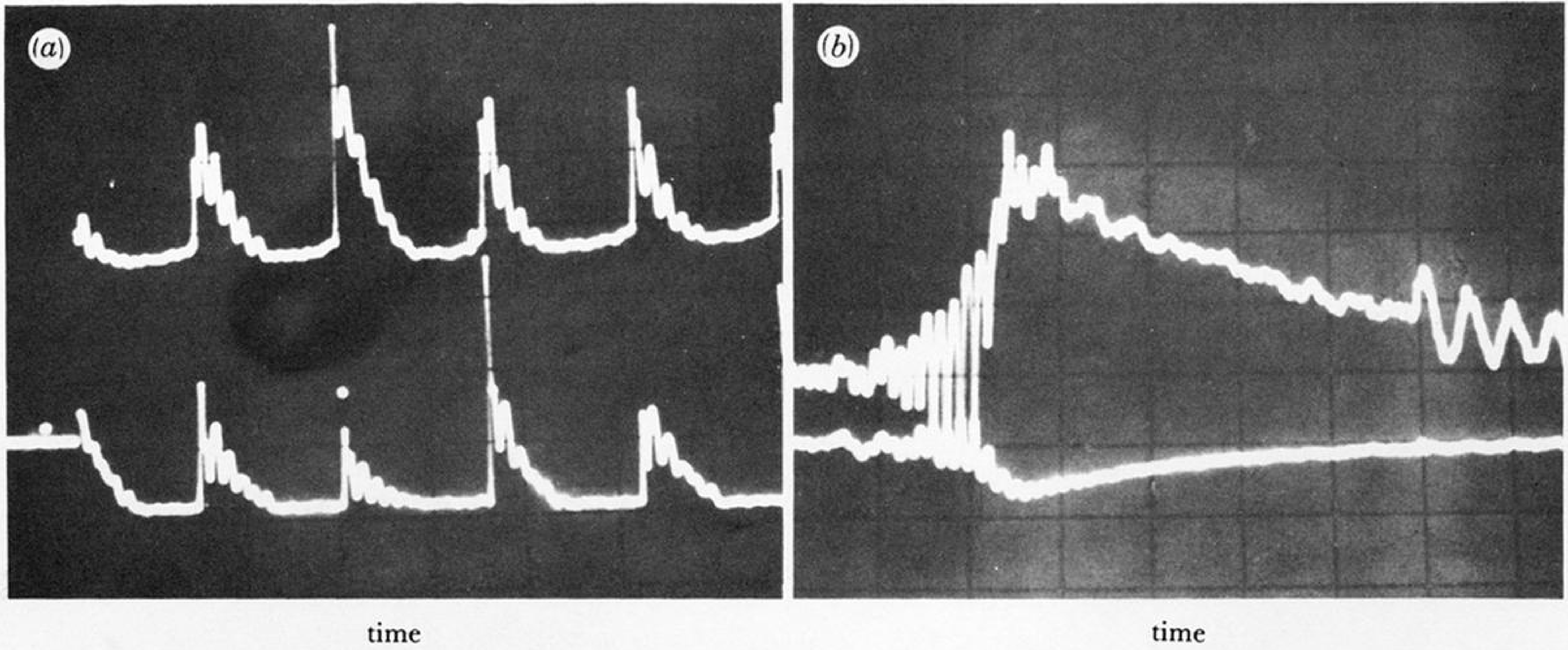

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